# Classical Thermodynamics Problem sheet 2

1. A heat engine operates between a body with finite heat capacity at initial temperature and a reservoir at fixed temperature . Show that the maximum amount of work that can be done is given by where:

Show that the work done is given by where the change in internal energy and entropy of the body are and respectively (it may be that you did this in order to show the above result – which is fine!).

If the body consists of 1 kg of water (heat capacity 4190 J kg-1 K-1), and , then how much work is done? (be careful with this information!)

1. Calculate the changes in the total entropy of the universe as a consequence of the following:
   1. A copper block of mass 1 kg, temperature 100 °C and specific heat capacity equal to 0.4 J g-1 K-1 is placed in a lake of large thermal capacity at 10 °C.
   2. An equivalent block at 10 °C is dropped into the lake from a height of 10 m.
   3. Two such copper blocks, one at 0 °C and one at 100 °C, are joined together.

You may assume that is independent of temperature.

1. For a material that has a magnetisation of magnitude *m* in a magnetic field of magnitude *B*, justify the statement that the internal energy *U* satisfies the following:

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Hint: variables *B* and *m* may be combined to produce a term giving the work done on the sample upon magnetizing it; consider which of these is an extensive and which is an intensive variable.

1. The Gibbs free energy *G* may be written in standard notation as

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Obtain a simplified differential expression for d*G*.

1. Thus, show that an analogue of the usual Clausius-Clapeyron equation that describes the phase boundary (in the *B-T* plane) separating two different magnetic phases, labelled 1 and 2, is

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(you may assume that *B* and *m* are collinear here).

1. Two identical bodies, each characterised by a heat capacity at constant pressure which is independent of temperature, are used as heat source and sink for a heat engine. The bodies remain at constant pressure and undergo no change of phase. Initially, their temperatures are and respectively; finally, as a result of the operation of the heat engine, the bodies attain a common final temperature .
   1. Prove that the total maximum amount of work done by the engine is:
   2. Use arguments based on entropy considerations to derive an inequality relating to the initial temperatures and .
   3. For given initial temperatures and , show that the maximum amount of work obtainable from the engine is given by
2. Show that the entropy change of a body (heat capacity *cp*) that is raised in temperature from T*i* to T*f* at constant pressure is given by

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1. Show that if two bodies of the same heat capacity but different initial temperatures T1 and T2 are placed in contact (but are otherwise isolated) then the total change in entropy when they have reached equilibrium is given by

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1. Although we’ve ignored this fact up to now, the heat capacity of solids is generally a function of temperature and, for metals, it is often written approximately as



where ** and *A* are constants. Show that the amount of heat required to raise the temperature of a metal body described by this approximation from temperature T1 to a higher temperature T2 is given by:



where you may assume thermal expansion is negligible.

1. Explain, using your knowledge of thermodynamics, why a helium balloon shrinks minutes after it’s been inflated from a He gas cylinder.
2. The latent heat of vaporisation for water at 100 °C is *L*v = 2250 kJ/kg. For one mole of water calculate (you may assume a density of water of 1 g/cm3 and that water vapor behaves like an ideal gas; the molar mass of water is 18 g/mol).
   1. the work done during evaporation,
   2. the heat released,
   3. *H,*
   4. *U,*
   5. *S,*
   6. *G.*
3. Use the Clausius-Clapeyron equation to show that the coexistence curve for the liquid-gas transition of one mole of substance is given by

where L is the latent heat of vaporisation. You may assume the vapour phase behaves perfectly.